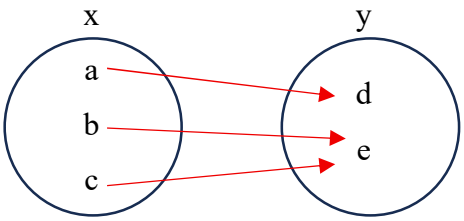
Chapter 2: Functions

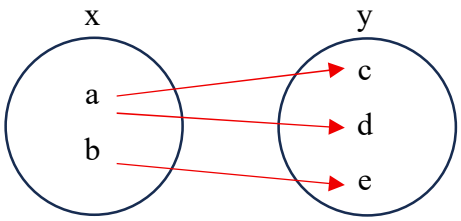
2.1 What is a Function?

In math, a function is a rule that pairs each input to exactly one output.

Functions can be represent several ways. One circle diagram below shows a function and the other does not.

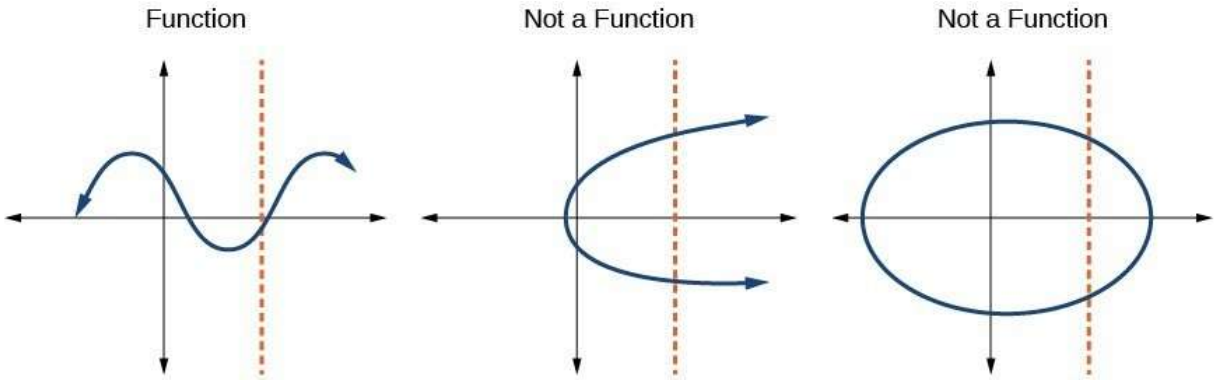


Function



Not a function

Functions can be graphed. To determine if a graph is a function, use the **Vertical Line Test**. Draw a vertical line anywhere on the graph. If a vertical line crosses a curve more than one time, it is not a function. If every possible vertical line crosses the curve at only one point (or not at all), the graph is a function.



2.2 Domain and Range

The domain of a function is its input. This is the set of values that can be put into the function. The letter that represents the input is called the independent variable because you can change it and see its effect.

The range of the function is the output, or the dependent variable. This is the set of values that the functions produces when the values of the domain are used in the function.

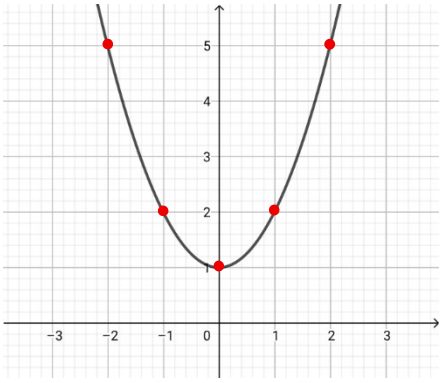
The horizontal axis represents the domain of the function, and the vertical axis represents the range.

2.3 Graphing Functions

To create a graph of a function, make a table of x and y values. Substitute each value of x into the function and calculate its y-value. This creates a table of x and y coordinates. Plot the coordinates on a grid and connect the points with a smooth line or curve.

Example: $y = x^2 + 1$

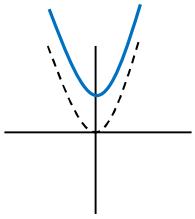
x	y	
-2	$(-2)^2 + 1 = 5$	(-2, 5)
-1	$(-1)^2 + 1 = 2$	(-1, 2)
0	$0^2 + 1 = 1$	(0, 1)
1	$1^2 + 1 = 2$	(1, 2)
2	$2^2 + 1 = 5$	(2, 5)



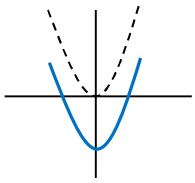
2.4 Transformations

When the shape of a function stays the same, but it moved up, down, left, right, or "flipped over, it's called a transformation.

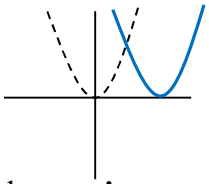
Vertical shift c units **up**:
Add a value to the function
$y = f(x) + c$



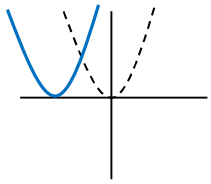
Vertical shift c units **down**:
Subtract a value from the function
$y = f(x) - c$



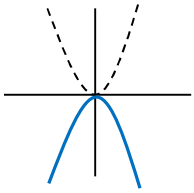
Horizontal shift c units to the **right**:
Subtract a value from x
$y = f(x-c)$



Horizontal shift c units to the **left**:
Add a value to x
$y = f(x+c)$

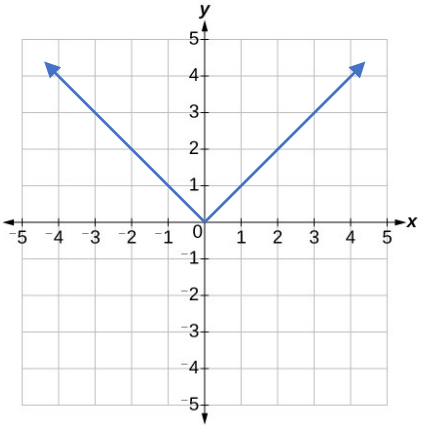


Reflection about the **x-axis**:
Multiply the function by -1
$y = -f(x)$

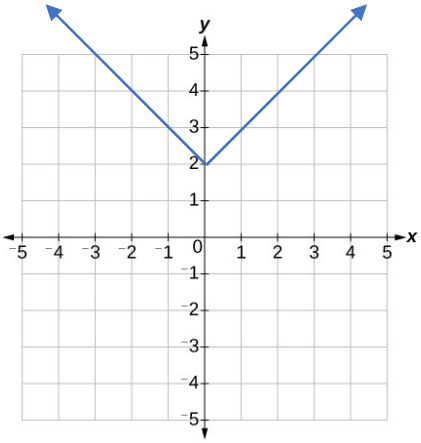


Example: For the function shown here, several transformations of the function are shown below.

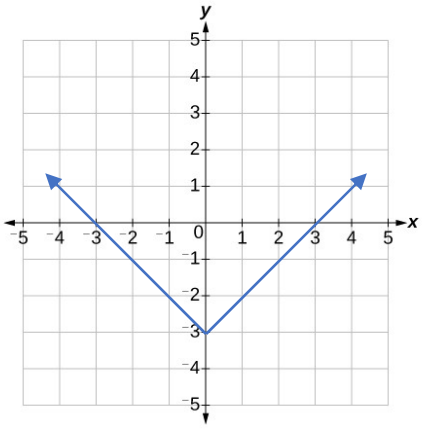
$y = |x|$



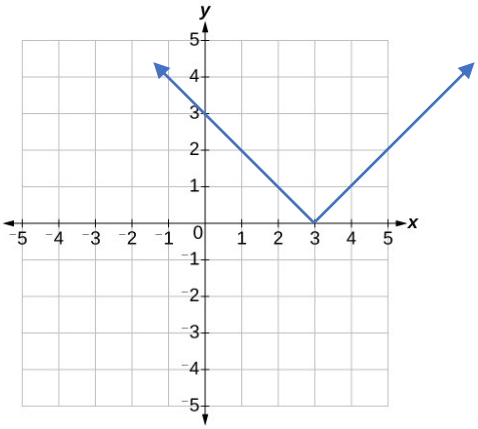
$y = |x| + 2$



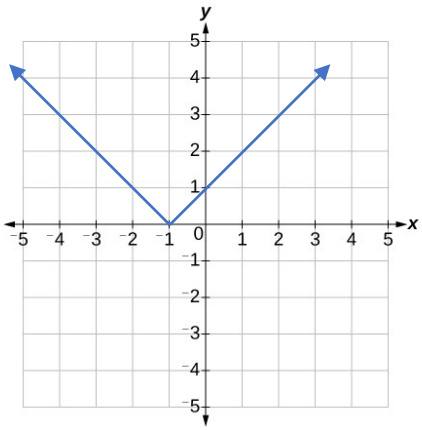
$y = |x| - 3$



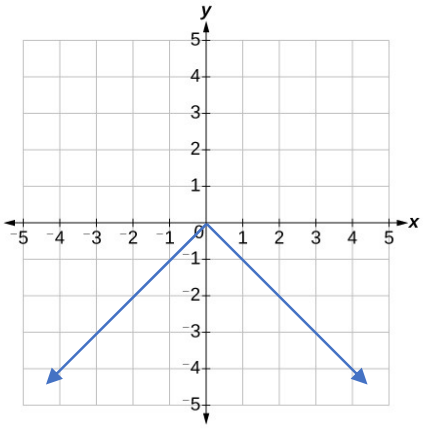
$y = |x - 3|$



$y = |x + 1|$



$y = -|x|$

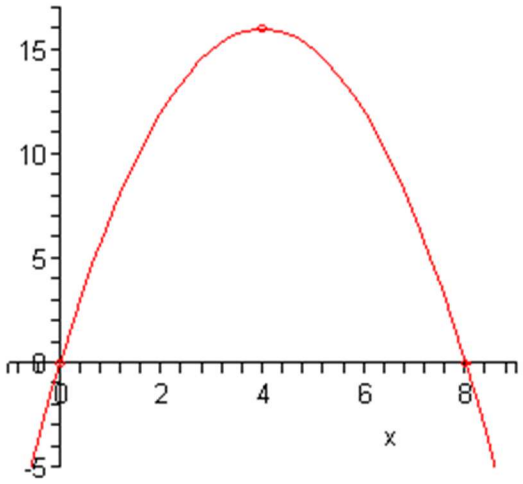


2.5 Real-World Functions

They say “a picture is worth a thousand word”. Sometimes it’s easier to understand things if we see a picture or diagram of it.

Real-World Functions

Functions of real-world events come in all shapes. Their domains and ranges have to reflect data that can actually occur. When you kick a football, the path of the ball can be represented by a parabola. The graph draw below could represent the path of the football, but would the football go underground? The graph continues below the x-axis. If this modeled a real football, that would be telling you that the ball traveled underground. Also, the ball is below the ground before it gets kicked!



To make the graph represent real events, we would set its domain to [0, 8]. This would limit the range to [0, 16].

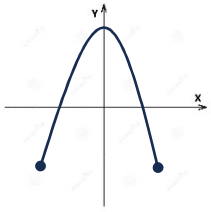
The variable on the x-axis is the independent variable. It is the “input” into the function and it affects the “output”.

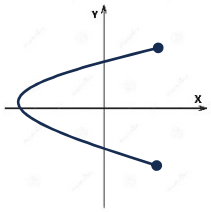
The variable on the y-axis is the dependent variable. Its value depends on what the x-value is.

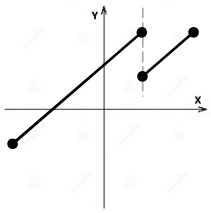
PRACTICE

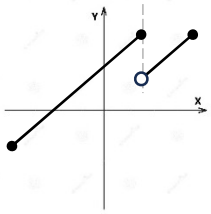
Section 2.1

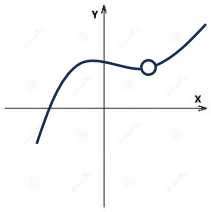
Tell whether each picture shows a function or not.

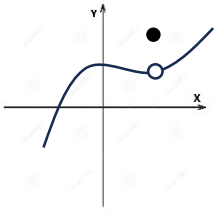
1. 

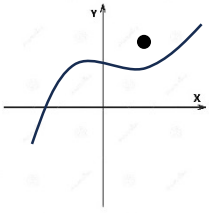
2. 

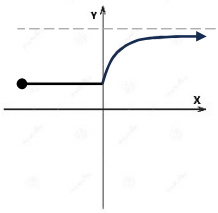
3. 

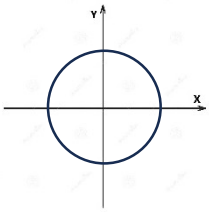
4. 

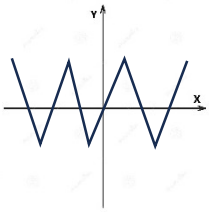
5. 

6. 

7. 

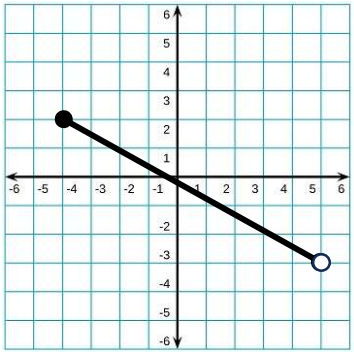
8. 

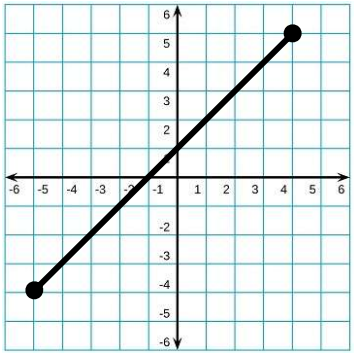
9. 

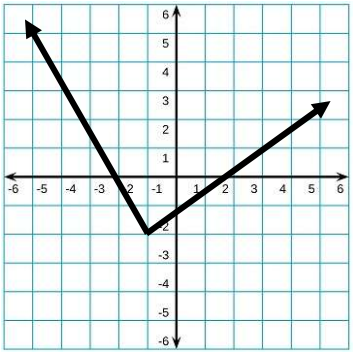
10. 

Section 2.2

Tell the domain and range of each function. Use interval notation.

11. 

12. 

13. 

14.