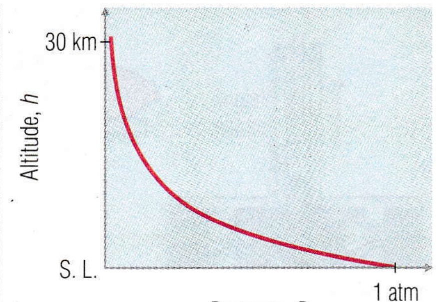
Ignoring the effect of temperature on the density of the atmosphere, the pressure at the top of Mt. Everest is about one-third of sea level pressure. This lack of air pressure at high altitudes causes *anoxia*, lack of oxygen, in mountain climbers who climb without supplemental oxygen tanks. It is also a problem for athletes who have trained at elevations near sea level but must compete at high-altitude locations.

Observe that all units in the complicated exponent in the exponential expression cancel out—exponents are dimensionless numbers.

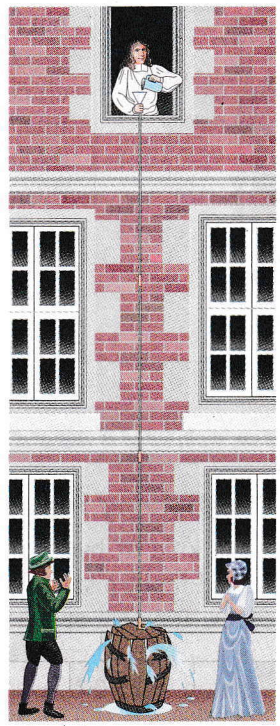


17-7 Atmospheric pressure varies exponentially with altitude. Blaise Pascal predicted that a vacuum existed above the atmosphere after he measured a decrease of atmospheric pressure with a barometer as he ascended a mountain.

17.6 Hydraulic Devices

Fill a sports bottle completely to the top. After tightening the cap with the pop top open, squeeze the bottle. What happens? The stream of water squirting from the cap indicates that the force of your hand was somehow transmitted directly to the mass of the water being ejected from the bottle.

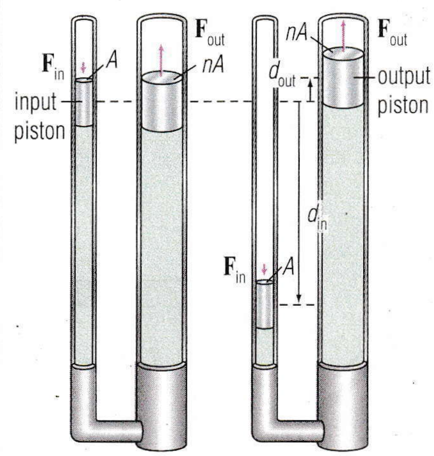
In the seventeenth century, Blaise Pascal studied the effect of pressure on incompressible fluids (liquids) that completely filled their containers. His experiments led him to the principle that is now called **Pascal's principle**: *the external pressure applied to a completely enclosed incompressible fluid is distributed in all directions throughout the fluid.* Consider Equation 17.6 again. If we shut the cap of the sports bottle and then squeeze on the bottle to change P_d, the reference pressure P_{ref} must increase by the same amount in order to maintain the equality.



17-8 In a famous experiment, Blaise Pascal attached a long, narrow pipe to a tightly sealed cask. After he filled the cask with water, the relatively small amount of water needed to fill the tube provided enough pressure to break the cask.

Blaise Pascal (1623–62) was a French mathematician, physicist, and philosopher known for his theory of probability and his work on pressure. He also invented one of the first calculating machines. The SI unit of pressure and a third-generation computer language are named after him.

Pascal's principle provides the theoretical basis for **hydraulic devices**, which are machines that transmit forces via enclosed liquids. Figure 17-9 shows how a small input force can generate a large output force. Since we are considering the *change* in pressure caused by an applied force, we can ignore the static pressure due to liquid depth throughout the device. Static pressure is the same at equal depths throughout the device and does not affect the change of output pressure and the associated force.



17-9 The principle behind hydraulic devices

The pressure created in the fluid by the force on the small piston is transmitted to the large piston. Assume that the cross-sectional area of the small piston is A and that of the large piston is nA. The pressure at the small piston is

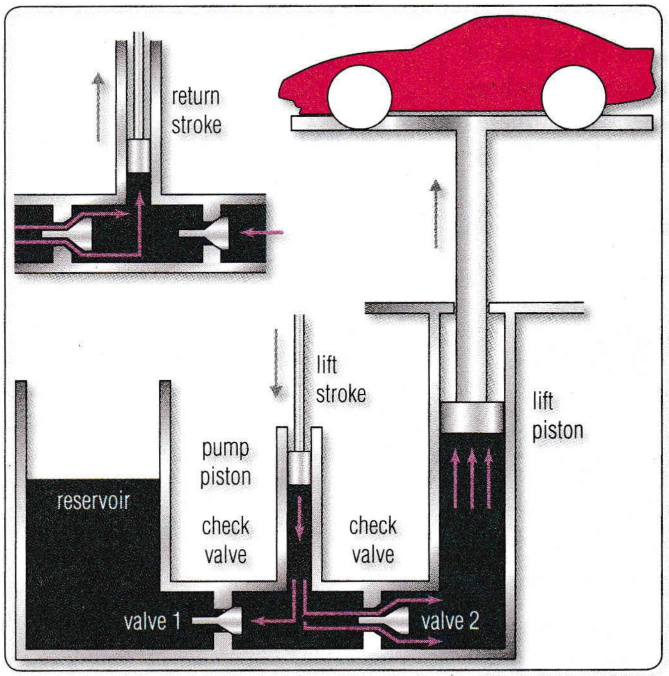
$$P = \frac{F_{in}}{A}. \quad (17.8)$$

Since the large piston is at the same height as the small piston, there is no effect on pressure due to height differences. Therefore, the pressure at the large piston is also P. The force on the large piston is

$$F_{out} = PnA.$$

Substituting Equation 17.8 for P, the areas cancel, giving the equation

$$F_{out} = nF_{in}. \quad (17.9)$$



17-10 A hydraulic lift

A small force on the small piston can lift a heavy load on the larger piston if the larger piston is big enough. For example, a small force on a hydraulic brake pedal can stop the wheels of a speeding car. Similarly, a child can lift a car by using a hydraulic jack.

It seems, at first glance, that a hydraulic device produces more energy than it consumes. In reality, the device simply transmits work. Mechanical work is the scalar product of force and displacement. The small piston experiences a smaller force, but it travels a greater distance than the larger piston. This is the distance principle you observed in simple mechanical machines. The work done on the large piston is equal to or smaller than the work done on the smaller piston. In real hydraulic devices, some work is used to slightly compress the fluid and overcome friction in the device, so less work is produced than is consumed.

EXAMPLE 17-3

Multiplying Force: A Hydraulic Jack

One kind of hydraulic jack uses water as its working fluid. The face of its small input piston is 10.0 cm above the bottom of the water reservoir in the jack. The area of the input piston is $A_{in} = 2.00\ cm^2$. The output piston's area is $A_{out} = 80.0\ cm^2$. (a) If the magnitude of the net force applied to the input piston is 25.0 N, what is the pressure at the *bottom* of the water reservoir? (b) What is the magnitude of the force exerted on the output piston?

Solution:

The pressure exerted by the input piston is computed from Equation 17.8:

$$P = \frac{F_{in}}{A_{in}} = \frac{25.0\ N}{2.00\ \cancel{cm^2}} \left| \frac{10^4\ \cancel{cm^2}}{1\ m^2} \right.$$

$$P = 1.25 \times 10^5\ N/m^2 \qquad (1)$$

a. Taking result (1) as P_{ref}, calculate the pressure at the bottom of the reservoir using Equation 17.6:

$$P_d = P_{ref} + \rho g d$$

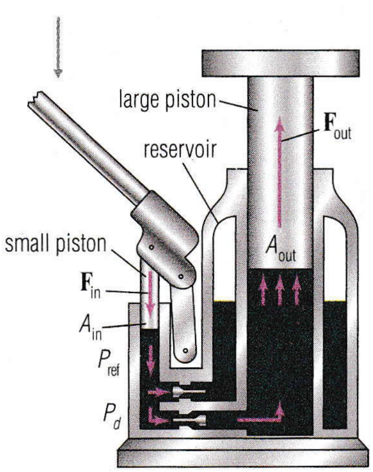
$$P_{d-10\ cm} = 1.25 \times 10^5\ N/m^2 + (1.00 \times 10^3\ \tfrac{kg}{m^3})(-9.81\ \tfrac{m}{s^2})(-0.100\ m)$$

$$P_{d-10\ cm} \cong 1.2\underline{5}9 \times 10^5\ N/m^2 \cong 1.26 \times 10^5\ N/m^2 \qquad (2)$$

b. Pressure at the same height in a closed container is equal everywhere. Therefore, no pressure adjustment for height is required. The ratio of the output piston's surface area to the input piston's is

$$n = \frac{A_{out}}{A_{in}} = \frac{80.0\ \cancel{cm^2}}{2.00\ \cancel{cm^2}}$$

$$n = 40.0.$$



17-11 Schematic of a typical manual hydraulic jack

Use Equation 17.9 to calculate the output force (assuming equal height of the piston faces):

$$F_{out} = nF_{in} = (40.0)(25.0\ N)$$

$$F_{out} = 1.00 \times 10^3\ N$$

The output force exerted by the jack is 40 times the input force.

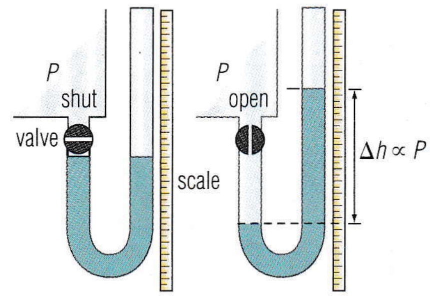
Problem-Solving Strategy 17.2

Under normal circumstances, the difference of pressure due to height differences between input and output pistons in hydraulic devices is negligible compared to the force transmitted by the hydraulic system.

17.7 Liquid-Medium Pressure Indicators

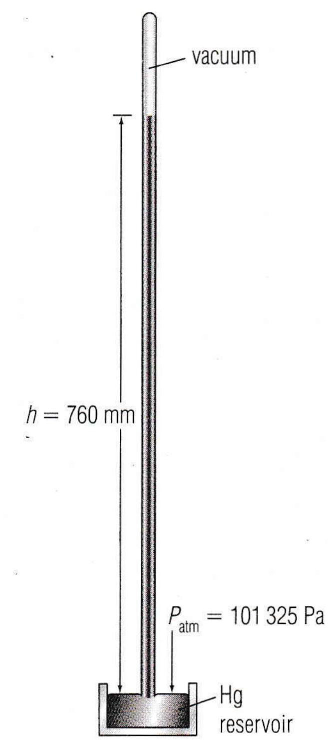
It is often important to know a fluid's pressure within a system, especially in engineering applications. For example, the crankcase of a diesel engine is a sealed chamber at the bottom of the engine, containing the crankshaft and the engine's lubricating oil. Operators need to know if a significant pressure is building up in the crankcase, which would indicate that fuel and exhaust vapors are accumulating and could cause an explosion. This pressure buildup can be caused by worn piston rings. A device called a **manometer** can be connected to the crankcase to indicate a positive or negative pressure compared to atmospheric pressure.

A manometer is a U-shaped transparent tube that contains a dense liquid such as mercury or some other nonvolatile liquid. One end is attached to the container of the fluid whose pressure is unknown, and the other end is open to the atmosphere. A flexible tube connects the two ends of the instrument. Before the fluid pressure is applied, the movable end of the manometer is adjusted until the level of liquid in both ends is the same. Then, system pressure is applied to the closed end of the manometer. Assuming that the system pressure is greater than atmospheric, the liquid level will drop in the closed end and rise in the open end of the manometer. The difference in the levels of the liquid on the two sides of the tube is proportional to the difference between the pressure in the system and that in the atmosphere. This difference of levels is read as a pressure on an attached scale. Obviously, if the monitored system's pressure is less than atmospheric pressure, then the liquid will rise in the closed end and drop in the open end of the manometer.



17-12 A manometer measures relative pressure.

The first instrument to accurately measure atmospheric pressure was the mercury **barometer**. This is essentially a long, straight glass tube sealed at one end, filled with mercury, and carefully overturned into an open reservoir of mercury so that no air enters the tube. If the tube is sufficiently long, the mercury meniscus at the top of the tube will drop until the atmospheric pressure exerted on the reservoir just balances the weight of the column of mercury. Greater atmospheric pressure supports a higher column of mercury. Standard atmospheric pressure, 101 325 Pa, supports a 760 mm column of mercury. As mentioned earlier in this chapter, these relationships provide various systems of units for reporting atmospheric pressure.

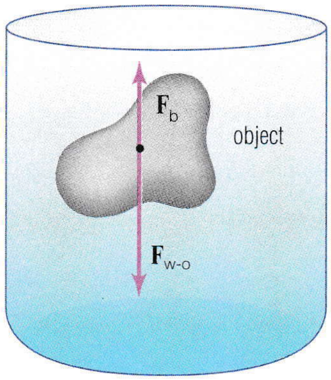


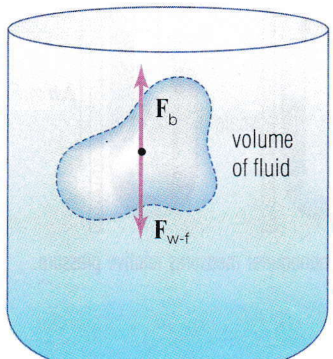
17-13 A mercury barometer

17.8 Buoyancy

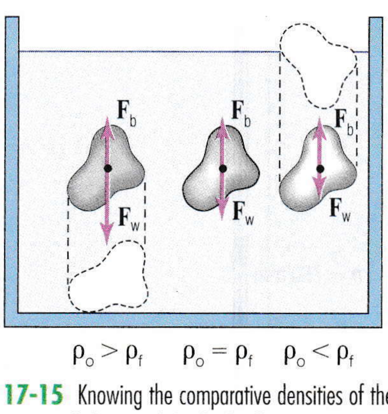
A story goes that, more than two hundred years before the birth of Christ, the scientist and mathematician Archimedes was given an assignment from his king, Hieron II. The king had ordered a goldsmith to make him a crown from the gold the king provided for him. The goldsmith made the crown and delivered it to the king. Then the king began to doubt the craftsman's honesty. Had he used some silver in the crown and kept some of the gold for himself? The king immediately weighed the crown and found that it weighed the same as the gold he had given the goldsmith. But what if the craftsman had replaced the gold with an equal weight of silver? Was there any way to find out? This was the problem the king took to Archimedes.

Archimedes thought hard about the gold crown. In what way do silver and gold differ, other than in value? Of course, their densities are different, but Archimedes did not have instruments to measure density accurately. Then one day the scientist was relaxing at the local bath. As he lowered himself into the full bath, some water spilled over the edge. Suddenly Archimedes had the answer to the crown problem. If he placed the crown in a full container of water, the amount of water that spilled out would depend on the crown's volume. A given weight of gold would force out less water than would a gold-silver mixture of the same weight. When Archimedes tested the crown, he found that it indeed was a gold-silver mixture. Thinking about the solution led Archimedes to the principle that the buoyant force of water on an object equals the weight of the water it displaces.





17-14 Buoyancy is the force a fluid exerts on an immersed object.



17-15 Knowing the comparative densities of the immersed object and the fluid will determine whether the object sinks, floats, or is neutrally buoyant.

17.9 Submerged Objects

What happens to an object placed in a fluid? Let's consider a volume of fluid to be the system. Its shape doesn't matter as long as it is in equilibrium with the surrounding fluid. The forces acting on the fluid-system are its weight ($\mathbf{F}_{w\text{-}f}$) and its equilibrant force, which we shall call the **buoyant force** ($\mathbf{F}_b$). The buoyant force results from the fluid pressure on the system. Because the fluid system is in equilibrium, these two forces are equal in magnitude but opposite to each other.

If the fluid system is now replaced by an object that is exactly the same shape, the gravitational force on the new object-system ($\mathbf{F}_{w\text{-}o}$) will be different. However, the buoyant force that is dependent on the pressure of the fluid on the *volume* of the object remains the same. The object experiences an upward force that is equal to the weight of the fluid it displaces. The magnitude of the buoyant force is determined from the equation

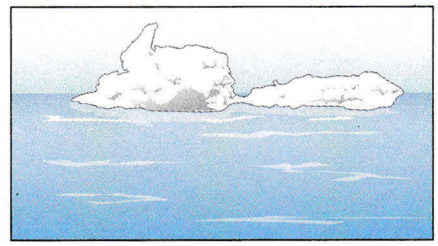
$$F_b = \rho|g|V, \qquad (17.10)$$

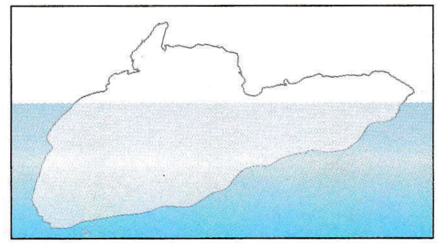
where ρ is the density of the *displaced fluid*. The fluid-system experienced the same force, but it was balanced by its own weight. Equation 17.10 is the mathematical expression for **Archimedes' principle**. This principle states that *any system that is submerged or floats in a fluid is acted on by an upward buoyant force equal in magnitude to the weight of the fluid it displaces.*

Archimedes' principle holds for a system of any shape. The buoyant force may be greater than a system's weight, less than its weight, or equal to its weight. If it is equal to the system's weight, the system behaves exactly like the fluid it displaces. It is not accelerated at all because the forces acting on it are balanced. This implies that the overall density of the system must equal the fluid it displaces, because its volume and weight (and mass) are the same.

Similarly, if the weight of a system is greater than that of the displaced fluid, its density is greater than the fluid's. Its downward weight will exceed the upward buoyant force, and the object will begin to sink. However, the net force accelerating the system downward is the difference of the system's weight and the buoyant force. Consequently, the system appears to weigh less in the fluid. You can verify this fact by lowering a heavy stone into a pond. As it submerges, the stone seems to become lighter.

If the weight of an immersed system is less than that of the displaced fluid, its density is less than the fluid's. An object less dense than the fluid will not have enough weight to balance the buoyant force. The net force will accelerate the system upward in the fluid. If the fluid is a liquid, the system will eventually break through the surface. When this happens, the buoyant force on the system rapidly decreases as the volume of liquid that is displaced decreases. Eventually, the buoyant force equals the weight of the system, and the system stops rising with part above the liquid's surface and part below.





17-16 Because the density of icebergs ($\rho_{ice} = 0.92$ g/cm^3) is less than that of seawater ($\rho_{sw} = 1.03$ g/cm^3), icebergs float on the ocean. The visible portions are often huge compared to oceangoing vessels, yet only 11 percent of their entire volume is above the ocean's surface.

EXAMPLE 17-4

Floating Like a Brick: Buoyancy

A 1.00×10^3 cm^3 brick ($\rho = 1.74$ g/cm^3) is placed in mercury ($\rho = 13.6$ g/cm^3). (a) What is the buoyant force on the brick? (b) Will it float?

Solution:

a. Determine the magnitude of the buoyant force on the brick fully submerged in the mercury:

$$F_b = \rho|g|V$$

$$F_b = (13.6 \text{ g/cm}^3)(9.81 \text{ m/s}^2)(1.00 \times 10^3 \text{ cm}^3)$$

$$F_b \cong 13\underline{3}400 \text{ g·m/s}^2$$

Convert grams to kilograms in order to obtain newtons:

$$F_b \cong 133 \text{ kg·m/s}^2 = 133 \text{ N}$$

b. The magnitude of the brick's weight is as follows:

$$F_w = m|g| = (\rho V)|g|$$

$$F_w = (1.74 \text{ g/cm}^3)(1.00 \times 10^3 \text{ cm}^3)(9.81 \text{ m/s}^2)$$

$$F_w \cong 17\,\underline{0}60 \text{ g·m/s}^2 \cong 17.1 \text{ N}$$

The brick's weight is less than the maximum possible buoyant force of 133 N, so the brick will float in the mercury.

Problem-Solving Strategy 17.3

If you need to know if an object will float in a liquid, compare the densities of the object and the liquid. If the object's density is less than the liquid's density, it will float. Otherwise, it will submerge or even sink.

If the fluid is a gas and the system can rise without restriction, such as a balloon in the atmosphere, the gas density decreases with altitude, and thus the buoyant force decreases as well. Eventually, an altitude is reached at which the densities of the balloon and the displaced gas are equal, and the balloon stops rising.

17.10 Buoyancy of Real Objects

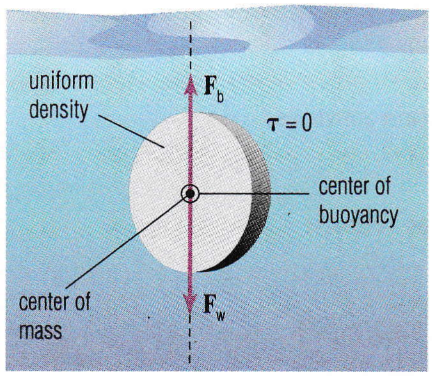
We have assumed that the submerged systems we have considered are rigid and have constant densities. These ideal systems do not respond to changes in fluid pressure as their depths or heights change. Real systems, such as helium balloons and submarines, *do* respond to pressure changes. For example, a submarine's crew can adjust the sub's weight at a specific depth so that its weight exactly equals its buoyant force—it is *neutrally buoyant.* If the submarine then moves to a shallower depth, the sea pressure becomes less and its hull expands slightly as the metal cylinder of the hull is less compressed. The submarine's volume then increases but its weight remains the same, so its density decreases. As a result, the buoyant force exceeds the weight of the submarine, and the net force acting on the ship is upward—it is *positively buoyant.* The opposite effect occurs if the ship goes deeper—it becomes *negatively buoyant.* Balloons and "rigid"

There are other fluid factors that can affect system buoyancy: variations in fluid density with depth and height, temperature changes, and changes of fluid composition. For example, seawater temperature and salinity significantly affect the buoyancy of a ship or submarine.

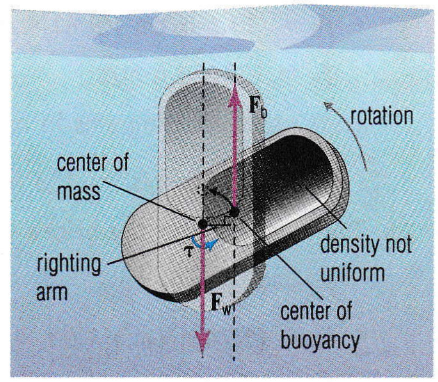
airships experience similar effects due to the variation in atmospheric pressure with altitude.

17.11 Center of Buoyancy

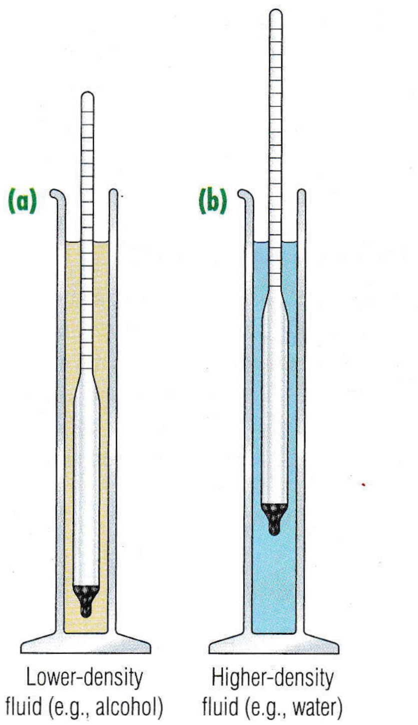
Every object submerged in a fluid has both a center of gravity (center of mass) and a center of buoyancy. The **center of buoyancy** is the center of mass of the fluid that would occupy the submerged space that the object occupies. If the object has uniform density and is completely submerged, its center of mass and center of buoyancy are the same. The same is true if the object's mass is distributed symmetrically. However, if an object's mass is not symmetrically distributed horizontally, its center of gravity will not coincide with the center of buoyancy. Therefore, the buoyant force and the gravitational force will not be on the same vertical line. A torque will exist on the object, and the object will rotate as it sinks, rises, or stays in place, until its center of gravity is on a vertical line with its center of buoyancy.



17-17a In a submerged object of uniform density and shape, the centers of mass and buoyancy are located at the same point.



17-17b When the mass is not distributed in a horizontally symmetrical position, the resulting torque will rotate the object until its centers of mass and buoyancy fall on the same vertical line.



17-18 Identical hydrometers in (a) alcohol and (b) water

Do not confuse *hydrometers* with *hygrometers*. The latter instrument is used to determine the relative humidity of the atmosphere.

17.12 The Hydrometer

The relationship between density and buoyancy provides a method for measuring fluid density. One instrument used to measure density is a **hydrometer**. The hydrometer is a cylinder with a weighted end. When it is placed in a low-density liquid, the hydrometer sinks more than it does in a high-density liquid. The volume of the liquid that the hydrometer must displace to float is inversely proportional to the density of the liquid. Each hydrometer is calibrated for use with a specific liquid. The scale is placed so that the surface of the liquid intersects it at the value corresponding to the liquid's density or its specific gravity. Hydrometers are commonly used to test the amount of antifreeze in a radiator and the concentration of electrolytes in a battery. Scientists use hydrometers to measure the amount of suspended sediment in lake and river water as well.