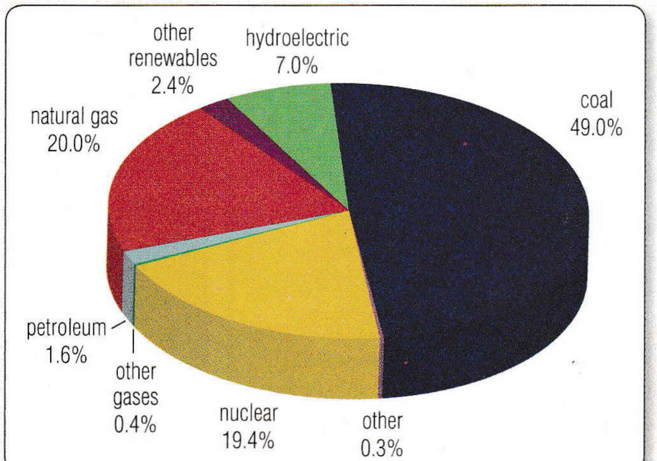
Fluid Mechanics 17

DOMINION SCIENCE PROBLEM

Homespun Energy

Nearly 70% of the domestic power generation needs of America are met using fossil fuels. Almost half of the fuel is coal, and most of the rest is natural gas. Though both energy sources are fairly economical, natural gas is slightly cheaper than coal and is now preferred for new power plant construction and upgrades. Very little electrical power is supplied by oil, which has never been cheap and has experienced significant jumps in price recently. Nuclear energy meets nearly 20% of the nation's energy demands, but its cost to the consumer is two to three times as much as that of natural gas. People still oppose nuclear power plants because they seem inherently dangerous and produce long-lived radioactive wastes. Hydroelectric power spans coal and nuclear power in cost, but its availability is limited by geography. Also, it is uncertain whether any new large power dams will be built in the United States. What safe, plentiful, and renewable alternative energy source could supplement or even replace the costly, nonrenewable ones in use today?



17-1 Relative contribution of power generation sources in the United States for 2006

Source: Energy Information Administration. *Electric Power Annual 2006.*

17A HYDROSTATICS: FLUIDS AT REST

17.1 The Science of Fluids

Objects can be immersed in fluids, and fluids can pass through objects. Fluids can flow around objects and structures, as well as flow through pipes and orifices. For these reasons, fluids exhibit some interesting mechanical properties that have great significance to the proper functioning of life processes as well as to strictly physical phenomena. In earlier chapters we discussed nonfluid mechanics—the mechanics of rigid objects. In this chapter you will study how Newtonian principles apply to fluids, a field called **fluid mechanics**. This area of mechanics can be broadly subdivided into **hydrostatics**—the study of stationary fluids in which all forces are in equilibrium—and **hydrodynamics**—the study of fluids in motion.

An orifice is a small opening in a barrier between two volumes through which a fluid may flow.

Recall from earlier chapters that fluids were defined as any matter that flows. Such substances include liquids, gases, and even some solid-looking matter such as the ice in glaciers.

17.2 Density

A major difference between the study of nonfluids and the study of fluids is that fluids are continuous. If you pour twenty cupfuls of water into a bucket, the water will assume one unified structure, but if you place twenty wooden blocks in a bucket, they will retain their own identities. Therefore, it is impractical to talk about the water and the wood in the same way. One consequence of this difference is that the mass of a fluid sample is less useful than the mass of a solid. The **density (ρ)** (mass per unit volume) of a fluid is usually more important in a physics context. Table 17-1 lists the densities of some common materials.

The Greek letter rho (ρ) represents **density**.

TABLE 17-1

Representative Densities (Liquids in Color)

Substance	Density (g/cm^3)
Hydrogen	0.00009
Nitrogen	0.00126
Air (dry)	0.00129
Oxygen	0.00143
Wood (balsa)	0.12
Wood (cork)	0.24
Wood (pine)	0.50
Gasoline	0.68
Wood (oak)	0.72
Ethyl alcohol	0.79
Oil (crude)	0.88
Ice	0.92
Oil (olive)	0.92
Water	1.00
Seawater	1.03
Blood (whole)	1.05
Table salt	2.17
Sand	2.32
Glass	2.40–2.80
Aluminum	2.70
Bromine	3.12
Iron/Steel	7.86
Copper	8.92
Silver	10.50
Lead	11.30
Mercury	13.60
Gold	19.30
Platinum	21.45

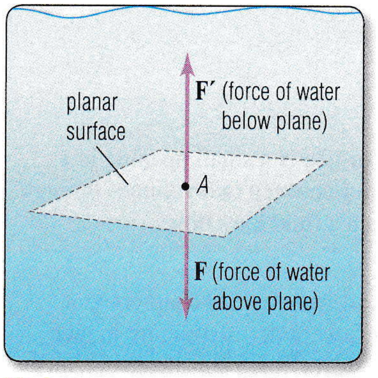
The densities in Table 17-1 are given in grams per cubic centimeter (g/cm^3), the common unit for density. The SI unit of density is kg/m^3. Water's density is 1000 kg/m^3. Since water's density is not numerically 1 in the SI, the densities of other substances in these units do not indicate their relative densities directly as they do in the common unit. However, if the densities of other substances are divided by the density of water, then we obtain a relative density called the **specific gravity (s.g.)**. The specific gravity of a substance is a dimensionless number that is numerically equal to the density of the substance in g/cm^3.

The **specific gravity (s.g.)** of a substance is the density of the substance divided by the density of water.

17.3 Units of Pressure

We have mentioned the concept of pressure in several contexts in earlier chapters. Fluid pressure is a distinctive property of liquids and gases. Perhaps you have experienced a change of pressure in your sinuses during a rapid descent or ascent on a mountain road or in an airplane. Skin and scuba divers do not have to go very deep before they experience significant pressure effects. How can we characterize pressure in a fluid?

Pressure is defined as the force exerted perpendicular to a unit area. If you select a small hypothetical planar area dividing two regions of fluid anywhere within the volume of the fluid (see Figure 17-2), the force exerted by the fluid on one side of the planar area is equal in magnitude but opposite to the force exerted by the fluid on the other side of the area. The two forces acting in opposite directions on different parts of the fluid form an action-reaction force pair and cancel each other according to Newton's third law. Consequently, at every point in a sample of fluid at rest the pressure at that point is equal in all directions. At the boundaries of the fluid, the container exerts a pressure on the fluid identical to the pressure the fluid exerts on the container.



17-2 The pressure on one side of a hypothetical plane surface is exactly equal to the pressure on the opposite side when the fluid is in mechanical equilibrium.

The SI unit of pressure is the pascal (Pa). Some other units of pressure are commonly used by different professions. The **atmosphere (atm)** was probably the earliest reference pressure. It is the average pressure of the atmosphere at sea level. In pascals,

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}.$$

While this is the accepted average value for atmospheric pressure, the actual value at a given location will vary due to factors such as elevation, weather, and temperature.

Chemists and other scientists often use the torr, named after Evangelista Torricelli, the inventor of the mercury barometer. This non-SI pressure unit is defined as $^1/_{760}$ of the pressure exerted by a column of mercury 760 mm high at sea level. This value was chosen because one atmosphere of pressure will support a 760 mm column of mercury in an evacuated glass tube, which was the basis for the first barometer. It should not be surprising that pressure may also be measured in mm of mercury or even inches of mercury. Atmospheric pressure is often reported by the popular media in these units.

Meteorologists are more likely to report atmospheric pressure in non-SI units called **bars** or **millibars (mb)**. A bar is 10^5 Pa, so

$$1 \text{ atm} = 1.013 \text{ bar} = 1013 \text{ mb}.$$

Millibars permit the convenient reporting of atmospheric pressures in whole numbers (or with only a single decimal place). A list of pressures equivalent to 1 atm, reported in various SI and non-SI units, is provided in Appendix C.

Engineers often report pressures in piping systems in **gauge pressure (P_g)**. Mechanical pressure gauges are designed so that one side of the pressure-detecting mechanism senses atmospheric pressure, and the other side senses system pressure. When the gauge indicates zero, it still experiences 1 atm. Therefore, the actual pressure, or **absolute pressure (P)**, in the system is the system gauge pressure plus 1 atm.

TABLE 17-2

Densities and Specific Gravities for Some Selected Substances

Substance	Density g/cm³	kg/m³	Specific Gravity
Air	0.001 29	1.29	0.001 29
Ethyl alcohol	0.79	791	0.791
Water	1.00	1000	1.00
Iron	7.86	7860	7.86
Mercury	13.60	13 600	13.60
Gold	19.30	19 300	19.30

Some news services and other sources of weather reports use inches of mercury for atmospheric pressure. 1 atm = 29.92 in. Hg.

The torr has no unit abbreviation. Some references give the unit for the torr as 1 torr = 1 mm Hg.

Evangelista Torricelli (1608–47) was an Italian mathematician and physicist. His many works included deriving principles in plane and solid geometry, describing ballistic motion (with Galileo), and demonstrating that a vacuum could be formed.

Most problems in this textbook will involve absolute pressure. However, in everyday life you will more likely read gauge pressure. The common system pressure unit in the United States is pounds per square inch (gauge), or psig.

17.4 Pressure in Incompressible Fluids

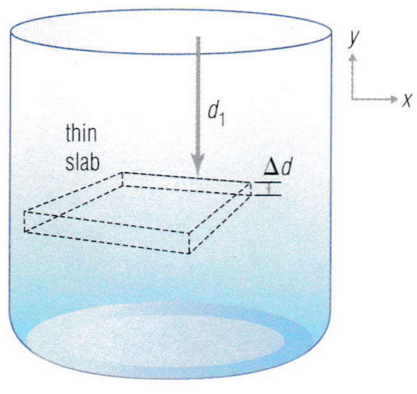
We have seen that pressure at a point in a fluid is the same in all directions. But we also know that pressure varies with the vertical position in the fluid. How does this occur? Let's first look at how pressure changes with depth in a liquid.

The particles in liquids are very close together, so they cannot be forced much closer together than they already are. For this reason, most liquids are essentially incompressible to a first-order approximation, so their densities can be assumed to be constant throughout the bulk of the liquid.

Let's choose a vertical coordinate system with the origin at the surface of the liquid so that $y_1 = 0$ m and positive is upward. Consider a small volume of the liquid, a thin rectangular slab with a square horizontal face of area A and thickness Δd. The top of the slab is at depth $y = d_1$ (a negative value) beneath the surface of the liquid. The liquid has a density of ρ, which is constant throughout the liquid. The pressure at the surface of the liquid is P_{ref}. The pressure at depth d_1 is P_{d_1}. The pressure of the liquid at the bottom of the slab is P_{d_2}, where the depth is

$$y = d_2 = d_1 + \Delta d.$$

(Note that Δd is a change of depth in the $-y$ direction.)



17-3 A thin slab of fluid at equilibrium

Assuming that the liquid is not in motion, then the slab is not moving, which means that the net force on the slab of liquid (the system) is zero. According to Newton's first law,

$$\Sigma \mathbf{F} = 0 \text{ N}.$$

Recall that

$$P = \frac{F}{A}, \text{ so}$$

$$F = PA.$$

The forces on the equal areas of the opposing sides of the slab are opposite and equal to each other, so they cancel and do not need to be considered further. The vertical forces acting on the slab of liquid must be in equilibrium as well. The downward force on the upper surface at depth d_1 is

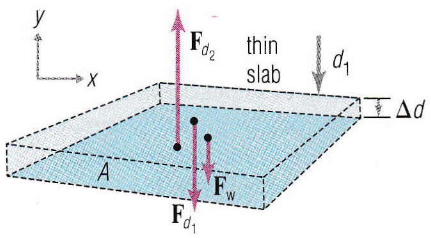
$$\mathbf{F}_{d_1} = -P_{d_1}A, \tag{17.1}$$

and the upward force on the lower surface of the slab at depth d is

$$\mathbf{F}_{d_2} = +P_{d_2}A. \tag{17.2}$$

There is one more vertical force to consider—the weight of the liquid in the slab itself,

$$\mathbf{F}_{\text{w}} = m\mathbf{g}.$$



17-4 Forces on the hypothetical slab of liquid

The mass of the liquid is determined by the product of the liquid's density and its volume:

$$m = \rho V$$

But the volume of the slab is the product of the area and the magnitude of the change in depth (Δd is negative):

$$V = A|\Delta d| = A(-\Delta d)$$

So, the weight of the mass of liquid is the expression

$$\mathbf{F}_{\text{w}} = \rho A\mathbf{g}(-\Delta d) = -\rho A\mathbf{g}\Delta d, \tag{17.3}$$

which is a downward force on the slab of liquid. Referring the forces to the vertical coordinate system where upward is positive, we have the force sum in the vertical direction.

$$\Sigma \mathbf{F}_y = \mathbf{F}_{d_1} + \mathbf{F}_{d_2} + \mathbf{F}_{\text{w}} = 0 \text{ N} \tag{17.4}$$

Substituting Equations 17.1, 17.2, and 17.3 for each force in Equation 17.4, we have

$$(-P_{d_1}A)_y + (P_{d_2}A)_y + (-\rho A g\Delta d)_y = 0 \text{ N}.$$

This is a one-dimensional problem, so we can dispense with the vector component notation. Setting the pressure terms equal to the weight term yields

$$-P_{d_1}A + P_{d_2}A = \rho A g\Delta d.$$

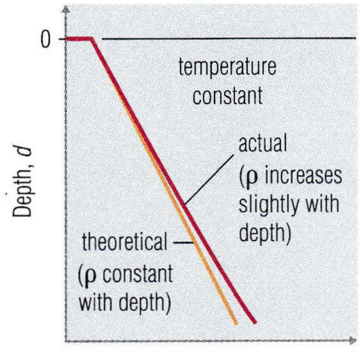
Dividing both sides by the area A and rearranging the left side yields

$$P_{d_2} - P_{d_1} = \rho g\Delta d. \tag{17.5}$$

Equation 17.5 indicates that the change of pressure from the top to the bottom of any slab of a liquid is a function of only the change of the depth, since liquid density and gravitational acceleration are assumed to be constant throughout the liquid (a reasonable assumption). Therefore, we can calculate the pressure at any depth d ($\Delta d = d - 0$) in a liquid using the following equation:

$$P_d = P_{ref} + \rho g d, \tag{17.6}$$

where d is expressed in *negative* scalar distances below the reference level at the surface of the liquid, and $g = -9.81\ m/s^2$. As you can see, pressure increases with fluid depth. If the container with the liquid is open to the atmosphere, the reference level d_1 is the liquid's surface, and the pressure $P_{ref} = P$ (the atmospheric pressure). For a closed container, P_{ref} may be greater or less than atmospheric pressure.



17-5 Pressure varies as an essentially linear function with depth. Compression of liquids at great depths causes the graph to depart from a truly linear relationship.

EXAMPLE 17-1

Ear Squeeze: A Practical Pressure-at-Depth Problem

Most people's eardrums can withstand a difference of up to 3.2×10^4 Pa between the external pressure and the middle ear pressure. This limitation can be a factor when free diving in a body of water. If a freshwater lake has a density of $1.00 \times 10^3\ kg/m^3$, how deep can an average person dive without fear of rupturing his eardrums? Assume that middle ear pressure remains at atmospheric pressure during the dive.

Solution:

We know that the maximum difference in pressure allowed (ΔP_{max}) is 3.2×10^4 Pa (1 Pa = 1 N/m^2). Let $P_{ref} = P$ = atmospheric pressure. Equation 17.6 will permit us to calculate d_{max}.

$$\Delta P_{max} = P_d - P_{ref}$$

$$\Delta P_{max} = (P + \rho g d_{max}) - P$$

$$\Delta P_{max} = \rho g d_{max}$$

$$d_{max} = \frac{\Delta P_{max}}{\rho g} \tag{1}$$

Substitute the known values into Equation (1) to solve for the maximum depth:

$$d_{max} = \frac{3.2 \times 10^4\ N/m^2}{(1.00 \times 10^3\ kg/m^3)(-9.81\ m/s^2)}$$

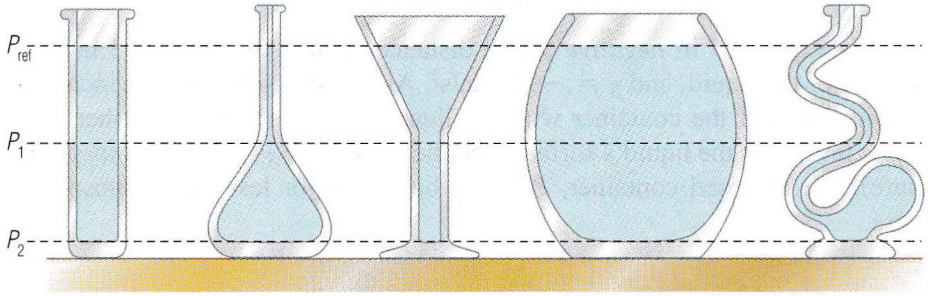
$$d_{max} \cong -3.\underline{2}6 \frac{N/m^2}{(kg/m^3)(m/s^2)} = -3.\underline{2}6 \left(\frac{kg \cdot m}{m^2 \cdot s^2}\right)\left(\frac{m^3}{kg}\right)\left(\frac{s^2}{m}\right)$$

$$d_{max} \cong -3.\underline{2}6\ m \cong -3.3\ m$$

The average person can free dive to a depth of about 3.3 m, or about 10 feet, without equalizing pressure in his ears and not rupture his eardrums. As a diver approaches this depth, a painful condition called *ear squeeze* occurs. Divers avoid this condition by forcing air into their middle ears via their eustachian tubes in order to equalize the pressure across their eardrums.

Change newtons to base SI units in order to complete the cancellation. Careful cancellation yields the expected unit for depth.

The shape and volume of a container has no bearing on the pressure at a given depth in a liquid. The apparatus in Figure 17-6 demonstrates this principle. The variously shaped containers have the same external pressure—the pressure of the surrounding air. When the liquid levels are the same in each container, the pressure is the same at the same depth.



17-6 Pascal's vases, illustrating the principle that fluids at the same depth have the same pressure

17.5 Pressure in Compressible Fluids

We saw in the previous discussion that the pressure at a given depth in an incompressible fluid is proportional to the depth and the density of the liquid above that depth. This is true for pressure in a gaseous fluid as well. However, the density is not constant with height (or depth, depending on the location of the reference point) in a gas because of a gas's inherent compressibility. In fact, gas density decreases approximately exponentially with height, so gas pressure varies the same way as long as other factors remain constant. (Recall from Chapter 14 that the density of an unconfined gas is also dependent on temperature as well as pressure.) In equation form, this relationship is

In Equation 17.7, *e* is the base of the natural logarithm. Your scientific calculator should provide a function that facilitates this calculation.

$$P = P_{ref}e^{-\frac{\rho_{ref}}{P_{ref}}|g|h}, \quad (17.7)$$

where P is the gas pressure at height h above the reference height, P_{ref} is the pressure at the reference height, and ρ_{ref} is the gas density at the reference height. Note that only the magnitude of g is used in the exponent.

EXAMPLE 17-2

Mountain Sickness: Pressure in a Compressible Fluid

What is atmospheric pressure at the top of Mt. Everest (8850 m)? Assume that pressure at sea level is 1.013×10^5 Pa, and atmospheric density does not depend on temperature (*not* a realistic assumption).

Solution:

You already have the information necessary to solve Equation 17.7 for pressure at 8850 m, or you can look it up.

$$P = P_{ref}e^{-\frac{\rho_{ref}}{P_{ref}}|g|h}$$

$$P = (1.013 \times 10^5\ \text{Pa})\exp\left[-\frac{1.29\ \text{kg/m}^3}{1.013 \times 10^5\ \text{N/m}^2}(9.81\ \text{m/s}^2)(8850\ \text{m})\right]$$

$$P = (1.013 \times 10^5\ \text{Pa})\exp\left[-1.105\ \left(\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{\text{m}^2\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(\frac{\text{m}}{\text{s}^2}\right)\text{m}\right]$$

$$P = (101\,\underline{3}00\ \text{Pa})e^{-1.105}$$

$$P \cong 33\,\underline{5}50\ \text{Pa} \cong 0.336 \times 10^5\ \text{Pa}$$

Problem-Solving Strategy 17.1

Complex exponential expressions are common in physics. In order to prevent confusion, the term "exp" is inserted before the expression in order to alert the reader that the following parenthetical statement is the exponent of the natural exponential base *e*. All units in the exponential expression must cancel.